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Correlating dynamic characteristics from field measurements and numerical analysis of a high-rise building

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SUMMARY

Using the concept of lumped masses and rigid floor slabs, several mathematical models were built using a popular PC-based finite element program to model a tall building with a frame-core wall structural system. These models were analysed to obtain the first nine mode shapes and their natural frequencies which were compared with those from field measurements, using numerical correlation indicators. The comparison shows several factors that can have a significant effect on the analysis results. Firstly, outriggers connecting the outer framed tube system to the inner core walled tube system have a significant effect on fundamental translational mode behaviour. Secondly, detailed modelling of the core considering major and minor openings as well as internal thin walls has the strongest influence on torsional behaviour, whose measurements were shown to be an important aspect of the dynamic behaviour for the structure studied. Fine tuning of an analytical model requires not just considering variation in values of structural parameters but also attention to fine detail. Copyright © 2000 John Wiley & Sons, Ltd.

KEY WORDS: structural dynamics; tall building; finite element modelling; model updating; structural analysis; vibration survey

DYNAMIC BEHAVIOUR OF TALL BUILDINGS

Dynamic behaviour is one of the most important design considerations for tall buildings; natural frequencies and corresponding mode shapes are basic data for seismic and wind response analyses. For example, the fundamental frequency is used to determine the seismic coefficient and site-structure resonance factor in the base shear formula used in the static approach of many earthquake codes e.g. UBC 1994 [1]. Furthermore, the mode shapes are used to determine the base overturning moments and vertical distribution of shear forces.

Although time-history analysis is not normally used in practical design of high-rise buildings, more and more important high-rise buildings requiring seismic design are analysed with this

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method to locate structural weakness. The accuracy of the analysis depends very much on the quality of the mathematical model used, and comparison or correlation of natural modes of the mathematical analysis with those of field measurements is a popular way to verify the credibility of the mathematical model.

STUDY TECHNIQUES FOR DYNAMIC RESPONSE OF TALL BUILDINGS

Two methods can be used to obtain the natural frequencies and corresponding mode shapes of a tall building. These are field measurements and analytical methods.

Field measurements

The most reliable estimates of natural modes are obtained from experimental measurements of an existing (prototype) building. Less reliable estimates can be obtained from a scaled physical model. For a prototype structure, experimental full-scale vibration test techniques include the very rare forced vibration test, where the response to a controlled and measurable dynamic load is recorded, and the more usual ambient vibration test (AVT) where response to natural or ambient loads (wind or micro-tremors) is recorded. Examples of such tests are reviewed by Ventura and Schuster [2] who also took the opportunity to use the AVT technique to determine the dynamic properties of a 30-storey building during its construction.

Analytical methods

It is very rare to have the combination of access to a building and a competent test team with the right equipment for field measurements and, of course, the prototype modes can only be identified when the building is complete. Thus it is more common and practical to use prior mathematical model analysis. This can be categorized into approximate theoretical analysis and finite element (FE) analysis.

Among the approximate theoretical methods, the finite strip method in structural analysis was presented by Cheung [3]. In order to overcome the problem of dealing with structures with irregular openings, a spline element was proposed by Fan and Long [4]. In this method, the element displacements are interpolated with spline functions. Highly accurate results could be achieved with lower-order functions and a few degrees of freedom.

Based on the fact that modal displacements at floor levels are linear combinations of displacement patterns due to different forms of deformation of a building structure, the displacement distribution factors method was developed by Leung *et al.* [5]. Compared with the distribution factors method, the finite storey method proposed by Pekau *et al.* [6] is based on nodal displacement fields obtained from two-storey substructures and intended to approximate shear, bending, and torsion components of global deformation. Both the distribution factors method and finite storey method reduce the number of unknowns to $15N$ and $5N$, respectively, for an N-storey building.

By replacing the tube with an equivalent rod, including the effect of the bending, transverse shear deformation, shear-lag and torsion, a simplified analytical method was proposed by Takabatake *et al.* [7] to do the preliminary design of doubly symmetric single and double frame-tubes in high-rise structures. Straight bars with variable cross-section were considered by Li *et al.* [8] in 1996 to study the static and dynamic analysis of multi-storey and high-rise structures.

Approximate theoretical techniques can be useful for preliminary studies but lack the ability to model fine details that are shown in this study to be crucial. These details can now be studied easily using PC-based FE software.

Correlation of analytical and field measurements

Full-scale testing of buildings is still quite rare and correlation with mathematical models rarer still. One remarkable example of a controlled systematic study on a full-scale model building is reported by Ellis and Ji [9]. Studies on correlating dynamic characteristics from field measurement and numerical analysis of tall buildings are seldom reported. Ventura and Schuster [2] developed five three-dimensional models of the 30-storey reinforced concrete building to correlate with their experimental data. Chajes *et al*. [10] developed a 47-element continuum model to perform an approximate 2-D dynamic analysis of a 47-storey steel-framed office tower and correlated the predictions for the first four vibration modes based on signals recorded during the 1989 Loma Prieta earthquake.

With ever increasing power of computers and ability to model to high levels of detail it is tempting to believe that this leads to more accurate analyses. Correlation studies including, as a bare minimum simple measurements of natural frequency, are the only means of confirming this.

STRUCTURAL SYSTEMS

Structural features of tall buildings

There are three common features for the modern structural system of a tall building:

(a) Boxed shear wall systems are well suited to regular plan office buildings [11]. In this kind of structural system, a stiff box-type structure is formed to group shear walls around service cores, elevator shafts, and stairwells. Since lateral stiffness requirements reduce in the upper portion of the building where low-rise elevators are terminated, a major opening often occurs on the core walls from the mid-height to the top. The core shear walls are usually penetrated by vertical rows of openings required for doors and corridors.

(b) With the increase in the building height and slenderness, core and outrigger systems are used, in which the outriggers have an inherent advantage in the efficient and economic design to reduce lateral deflections.

(c) Many panels of internal thin walls are necessary to separate the interior space into elevator shafts, stairwells, and service cores.

The contributions of coupled core walls, outriggers, and internal thin walls to the stiffness of the whole structural system and natural vibration modes are seldom reported. The purpose of this study has been to investigate their effects and present some guidance for modelling the structural system of tall buildings.

In this study, several FE models representing the three-dimensional (3-D) behaviour of a tall building have been created and analysed. Results from analysing models with various assumptions about structural members are compared with the field measurement results obtained by an AVT. The relationships between natural frequencies and structural characteristics are summarized.

Republic Plaza

The structural system studied here is the 280 m, sixty-six storey Republic Plaza tower, one of the three tallest buildings in Singapore. The tower has a frame-tube structural system with an internal core wall connected to a ring of external columns by horizontal steel framing system at every floor.

The reinforced concrete (RC) central core wall has a plan area of 21.5 m $\times$ 22.65 m and extends almost the full height of the building. Except for the top few levels, the core wall varies in thickness from 600 to 400 mm and contains an RC core slab at every level. Figure 1 shows the layout of the standard floor plan at level 18, and the elevation view of this building is shown in Figure 2. Much of the core is taken up by the numerous lift shafts. Note that a set of low rise (LR) lifts reaches only

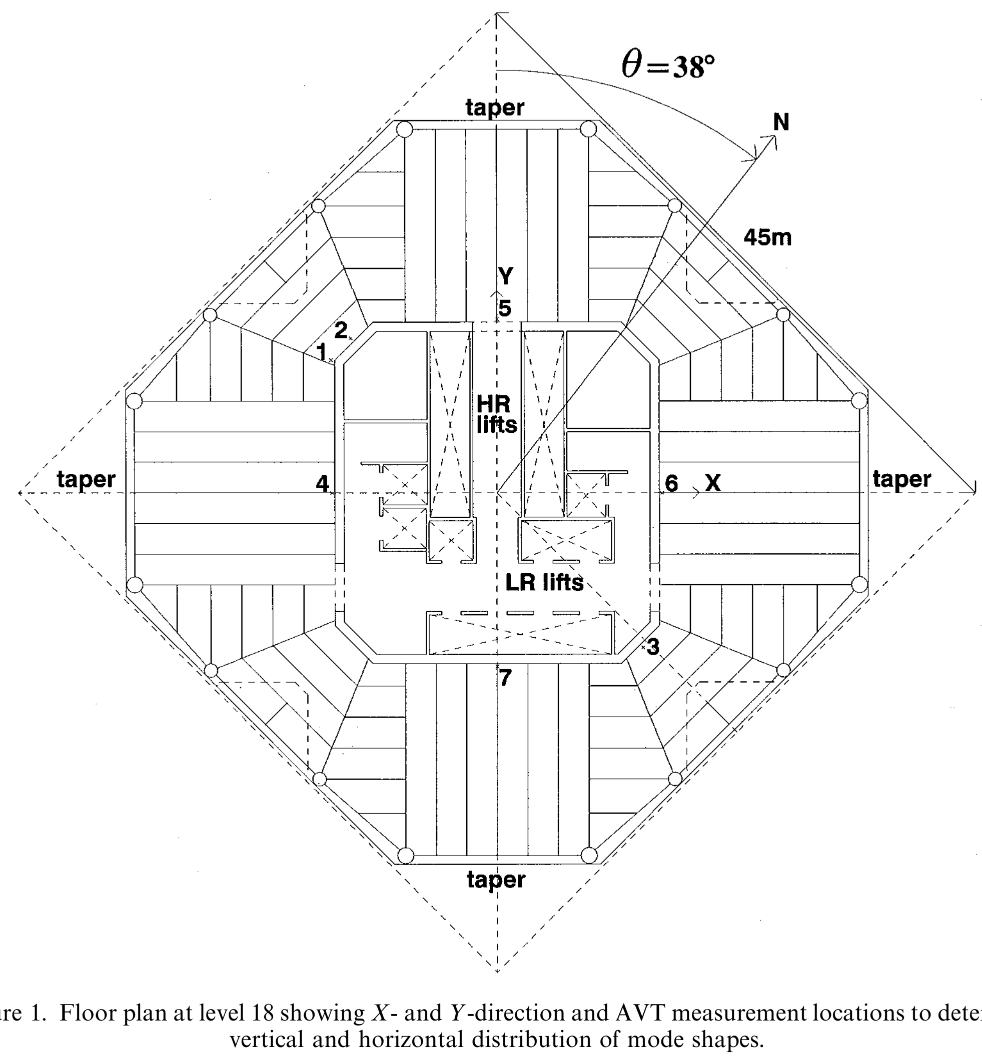

Figure 1. Floor plan at level 18 showing X- and Y-direction and AVT measurement locations to determine vertical and horizontal distribution of mode shapes.

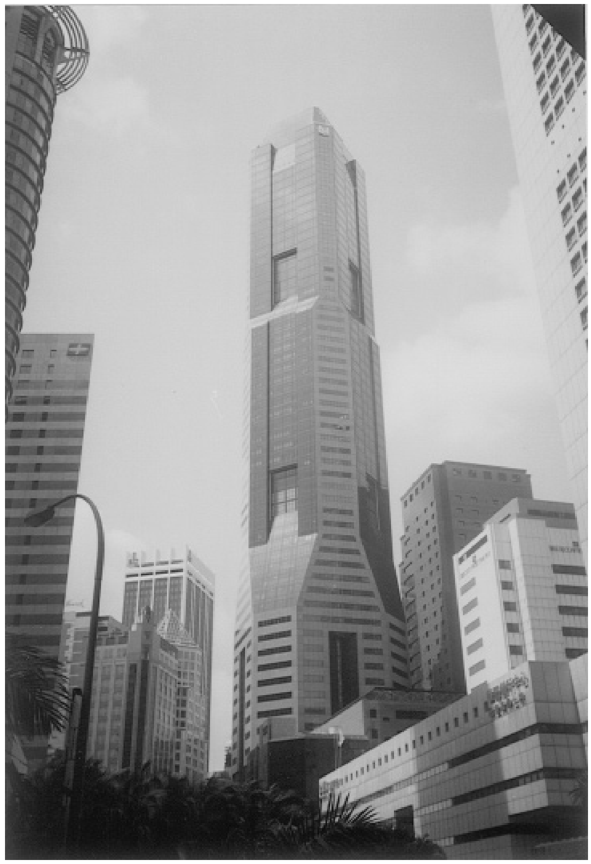

Figure 2. Vertical view of Republic Plaza, Singapore.

to the 35th floor, above which the core is open at that side from the 37th floor upwards, while other lifts extend to the highest floors.

The perimeter of the building comprises eight large steel tube columns with diameters up to 1.22 m diameter and eight smaller columns, up to 1.02 m diameter. These columns reduce in diameter at higher floors and up to the 49th level are filled with concrete. Up to level 62 the perimeter lies in a square with dimension 45 m and has two tapering sections between levels 20 and 27 and between levels 44 and 46. Two mechanical equipment floors are located at levels 28 and 47 above the tapering sections and have twice the normal storey height of 3.95 m. Outriggers were installed on these two levels to enhance the rigidity of the building frame under lateral loads. The connections of the outrigger systems were not made until the building was completed to avoid introducing dead loads into the bracing.

The horizontal framing system has moment resisting connections at the beam-column joints while the beam to core connections are simply pinned. The column bases are bolted to the foundation at basement level B1 where they sit on a deep stiff foundation system. The foundation system comprises six inner caissons founded up to 62 m deep in boulder clay and connected by

a 5.5 m thick concrete mat, and eight exterior caissons founded up to 40 m deep and linked by deep transfer beams. All caissons are 5 m in diameter.

EXPERIMENTAL PROCEDURES

An AVT of Republic Plaza was conducted in late 1995 [12] just after the structural system of the building was completed but before fitting out for tenants and installation of various non-structural elements. Prior to the AVT a series of measurements of static stress and strain and of fundamental natural frequencies had tracked the building performance over two years of its construction program [13].

The objectives of the AVT were to identify

(1) the first three or four translational modes along the major structural axes (labelled as X and Y directions in Figure 1) and
(2) torsion by measuring the horizontal components of vibrations (accelerations) at several levels in the building.

Frequencies were identified from peaks in the Fourier spectra of the ambient vibrations. Mode shape values were estimated by setting a reference accelerometer at the highest level (level 65) and moving three more accelerometers to various lower levels. By recording the simultaneous response at different levels and in both directions and comparing magnitudes and phases to the reference values at each of the modal frequencies, the vertical variations for each of the translational mode shapes were obtained. Strictly speaking, the recorded vibration shapes are known as "operating deflection shapes" since they are not the pure contribution of a single mode. Nevertheless, due to the low damping the difference is mostly negligible. In addition accelerometers were placed in turn at different positions and orientations at one level to determine the rotational behaviour at levels 65, 46, 34 and 18. From the combination of the vertical and plan-wise variations of the mode shapes a complete picture of the translation and rotation of the building in each mode was obtained for 12 modes:

four modes dominated by translation in the X-direction,
four modes dominated by translation in the Y-direction, and
four modes of mainly torsional response.

The 12 natural frequencies (in Hz) averaged over all the measurements are summarized in Table I.

Owing to the random excitation the frequency estimates differ by a fraction of a percent between data records; the frequency values are averaged estimates. Spectral estimates obtained from random loads are subject to quantifiable errors, provided the loads (and of course, the structural parameters) are stationary. The wind loading responsible for the measured response is characterized by a flat spectrum and for the response levels recorded structural behaviour is practically linear. Based on an ensemble average over 16 records used for each measurement, with spectral resolution of 0.004 Hz, the standard error (coefficient of variation) in a single spectral estimate would be 0.125 while for a curve-fitted resonant frequency f it would be $\sigma(f)/\mu(f) \approx 0.005$. Hence the frequency estimates are accurate to 0.5 per cent (one standard deviation). The mode shape ordinates are ratios of spectral estimates subject to the same errors and the factors affecting mode shape accuracy are signal-to-noise ratio, measured by the

Table I. Translational and torsional mode frequencies for unoccupied building.

Mode	Frequency/Hz					
	X-direction		Y-direction		Torsion	
1	$X1$	0.191	$Y1$	0.199	$T1$	0.566
2	$X2$	0.703	$Y2$	0.746	$T2$	1.340
3	$X3$	1.550	$Y3$	1.730	$T3$	2.310
4	$X4$	2.483	$Y4$	3.011	$T4$	3.330

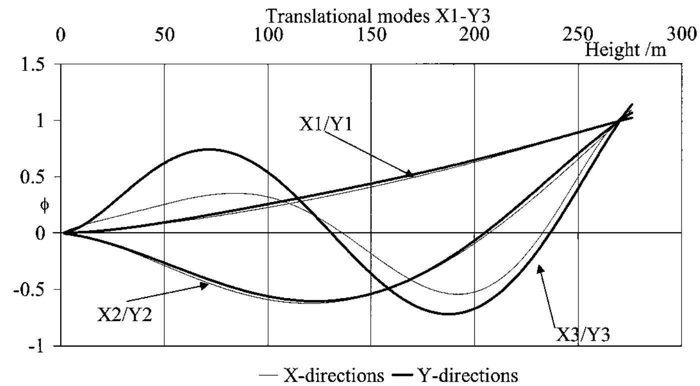

Figure 3. Translational mode shapes from field measurements.

coherence function. Coherence values for the measurements are close to unity except close to nodal points where contributions from other modes become most significant. Otherwise the ordinates are unbiased, and accurate to within 5 per cent at worst.

Smoothed vertical plane mode shapes for modes $X1$, $X2$, $X3$, $Y1$, $Y2$ and $Y3$ are shown together in Figure 3. For the X-direction modes the ordinates are components of building motion measured in the X direction ($\theta = 90^\circ$) at the same position with respect to the core wall (location 1, Figure 1) on various levels in the building. For the Y-direction modes the ordinates are for components at $\theta = 0^\circ$.

From examination of the various Fourier spectra of the measurements, it is known that some torsional modes are only weakly sensed in the X-direction at location 1. This can be explained by the asymmetry of the building which results in rotation about a position that is not at the geometric centre of the building plan but which lies close to the sensing axis of the accelerometer. Further, it is possible that translational modes have an element of rotation. In both cases it is possible to determine the plan-wise co-ordinates (x, y) with respect to the geometric vertical centreline of the building at which translation is zero. Such a location or 'centre of rotation' (COR) may not lie within the building plan and may vary from level to level and mode to mode.

Torsional mode shapes are shown in Figure 4(a). These are the components of translation ϕ_X, ϕ_Y measured in the X- and Y-direction at location 1 having co-ordinates $(-r, r)$ with respect to the building centreline. Figure 4(b) shows the variation of the co-ordinates (x, y) for modes $T1$

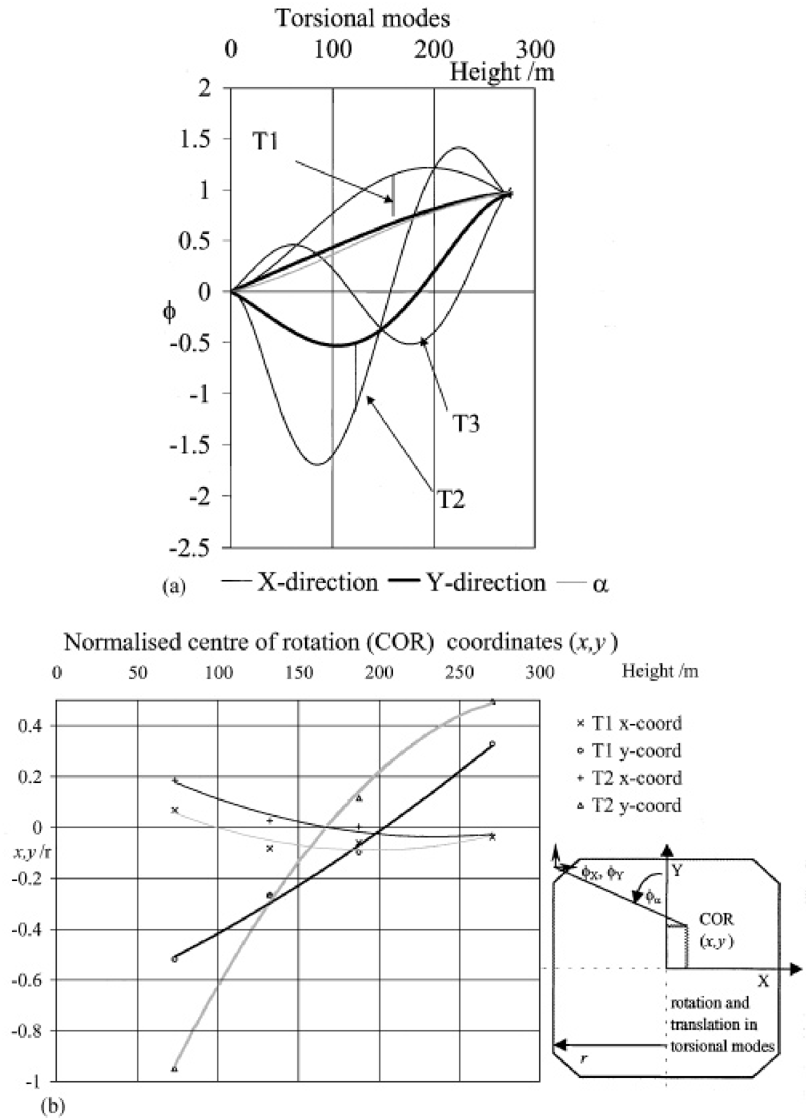

Figure 4. (a) Torsional modes determined from AVT. (b) Location of center of rotation (COR). $R =$ core wall semi-width ≈ 11 m.

and $T2$ at level 65. By interpolation between values of (x, y) known at four levels and using kinematic relationships between rotation and translation at a specific height in the building as shown in the inset of Figure 4(b) i.e.

$$\begin{aligned} \phi_X &= -(r - y)\,\phi_\alpha \\ \phi_Y &= -(r + x)\,\phi_\alpha \end{aligned} \tag{1}$$

the variation of torsional mode shapes ϕ_α can be plotted, as shown for mode $T1$ in Figure 4(a). The significant differences between ϕ_X, ϕ_Y for $T1$ and $T2$ show the strong influence of the varying location (x, y) given in Figure 4(b).

What Figure 4(b) shows is that the x-co-ordinate of the COR remains close to the geometric centreline, which is consistent with the symmetry of the building about the Y-axis as shown in Figure 1. The strong variation in y-co-ordinate of the COR is due to the asymmetry in core wall components resisting X-direction movement. At lower floors having the high-rise lift shafts the building rotates around a location below the X-axis, while for higher floors with the large opening replacing the lift shafts, the building rotates around a location above the X-axis.

Although modal ordinates at basement level could not be measured in the original testing, subsequent records obtained during strong winds have shown the fundamental mode ($A1$, $B1$) response modal ordinate at the basement to be 0.005 with respect to unit mode shape value at the roof. For $A2$ and $B2$ the figures are 0.01 and 0.026, respectively. These very low values, almost at the limit of instrument resolution, indicate a very stiff foundation consistent with the assumption made in the analysis.

ANALYSIS METHODS AND MODELS

Analytical models were based on the structural drawings and other information that was provided by the architect and the contractor. In addition, the contractor provided a detailed record of the actual masses of core wall, columns, core slab, office slab, curtain wall, and several water tanks at each storey during the construction period.

FEM1-stick model

The stick model (designated FEM1) shown in Figure 5 is the simplest model. Along the height of the building, each storey is modelled as one node point with six degrees of freedom (DOF). It has 66 frame elements and 402 DOF. Utilizing the technique of lumping masses, masses of the nth node/floor were determined as the sum of masses of core slab, office slab, and large water tanks at the nth floor, and the averaged mass of core wall, columns and curtain wall between the $(n-1)$th and nth storey. The mass moments of inertia about the z-axes were determined as the sum of those separate mass moments of inertia about the z-axis.

In this model, the framing system including all columns and beams is omitted, and the core wall is modelled as a frame element between each pair of nodal points on a vertical line. The flexural inertia values for X- and Y-direction bending (I_{xx} and I_{yy}) are calculated according to the standard bending formulae. The torsional rigidity J of the core wall is calculated as a multi-cell section with the following formula [14]:

$$J = 4A_o^2/\oint \mathrm{d}s/t \qquad (2)$$

where A_o is the area enclosed by the closed core wall, s is the periphery of the closed core wall, t is the thickness of the closed core wall.

The lower part of the building, from basement to level 37, is assumed as a closed thin-walled section, so the contribution of the internal walls to the torsional rigidity is relatively small and can be neglected. From level 38 to the roof, the core wall now has major openings, but two new closed

Figure 5. Stick model without considering frames.

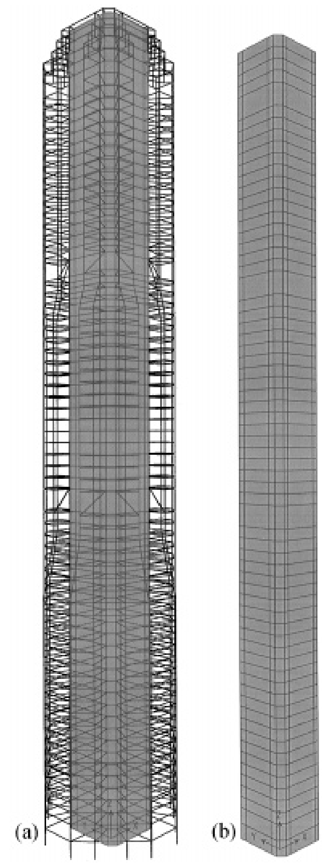

Figure 6. (a) The simplest 3-D model with a closed core wall and framing system. (b) The core wall component of FEM2.

thin-walled sections intersect and interact with the core wall, adding directly to the rigidity of the opened core wall. At basement level the core is assumed to be fully fixed to the foundation.

A two-dimensional (2-D) model in either of the apparent planes of symmetry (XZ or YZ) through the building can be used to capture the translation parts of the structural behaviour. The disadvantage of the 2-D model is that the spatial interaction cannot be considered and the analysis results give no prediction of the torsional structural behaviour, which cannot be ignored for dynamic response considerations.

With better computing facilities, complex 3-D models are now practical for analysis on a personal computer (PC). By lumping masses at FE model nodes and assuming rigid floor slabs, the PC-based FE code SAP2000 [15] was used to model the frame-core wall system of the building in several steps. For the following six 3-D models, the concept of master joints [15] was also applied such that at each floor, masses and mass moments of inertia of all structural elements including the rigid floor slab are lumped at one master joint. Their final values are the same as those of node points in the stick model (FEM1). It should be noted that lumped masses reflecting

all structural components are not changed among the different models, in order to study clearly the changes of lateral stiffness and torsional rigidity.

FEM2-model with closed core wall and frames

Figure 6(a) shows the simplest 3-D model with a closed core wall and Figure 6(b) shows the core wall component of the simplest 3-D model. The core walls are modelled as shell elements without considering any openings, taking Young's modulus E as 34 kN/mm^2 for the grade 50 concrete used. From level 1 to level 39, the thickness of the shell elements is set as 600 mm and at higher levels the thickness is taken as 400 mm. All columns, main beams, and 16 inclined outriggers are modelled as steel frames. As secondary horizontal floor beams contribute little to the stiffness of the whole structural system, they are omitted from the framing system in order to simplify the model. There are 3156 frame elements, 1056 shell elements and 18 960 DOF in this model.

FEM3-model with major openings and frames

FEM3 (with 3185 frame elements and 1027 shell elements) is developed from FEM2 by removing material for the major openings (10.9 m wide) from level 38 upwards as shown in the core wall (Figure 7). All the shell elements between the openings on the different levels with major openings are removed.

FEM4-model with major openings and secondary openings

In the real building, the core walls are usually perforated by the vertical lines of secondary openings that are required for doors and corridors, and which result in coupled shear walls. The dynamic structural behaviour of 2-D coupled shear walls has been studied by some researchers, e.g. by Li and Choo [16, 17] who used a continuous–discrete approach for the natural frequency analysis of 2-D coupled shear walls. However, few studies have been reported for the behaviour of 3-D coupled shear walls. In model FEM4 the coupled core walls are modeled by incorporating vertical lines of secondary openings as shown in Figure 8.

For every level, a standard opening 3.2 m wide and 3.25 m high is located in the middle of north side of each storey. In the northwest and northeast corners of the core wall, two smaller openings are used to approximate door spaces at each storey. In the southwest and southeast corners of the core wall, there are two additional standard openings from level 1 through level 37. For the convenience of modelling, and due to location of nodal points, these openings are all set as 2.65 m wide and 3.25 m high and the shell elements located at these locations are removed. Triangular and trapezoidal shell elements are used as transitions to rectangular shell elements. FEM4 has 3185 frame elements, 1959 shell elements, and 20778 DOF.

FEM5-model with major openings, secondary openings and internal thin walls

Figure 1 shows some of the many thin (200 mm) internal walls located within the core wall. These walls have relatively small translational stiffness due to their thickness and proximity to the neutral axis; so it seems that they could be omitted in the structural analysis. However, the torsional rigidity of a closed cross-section is much larger than that of a similar section with a partial opening. In this building the core wall can be assumed to be partially opened at the lines

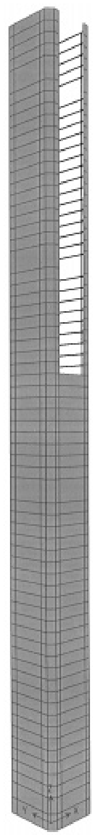

Figure 7. The core wall component with major opening of FEM3.

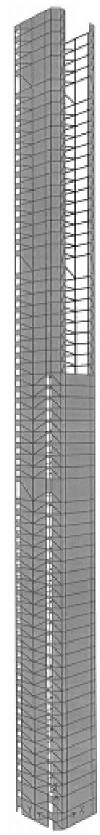

Figure 8. The core wall component with major opening and secondary openings of FEM4.

of secondary openings in addition to the major openings. However, when the internal thin walls are added, the core walls become partially closed. These thin walls could be useful for enhancing the torsional rigidity of the whole structural system. Due to the complexity of modeling, the four panels, approximating the internal thin walls, as shown in Figure 9 are added to FEM4 to form FEM5. FEM5 shown in Figure 10 thus represents the most complete structural model, which contains 3185 frame elements, 2223 shell elements, and 20 808 DOF.

FEM51-model FEM5 without outriggers

As shown in Figure 10 (for the FEM5 model with core wall and framing system), 16 steel beams are employed as outriggers at the two mechanical equipment floors to limit storey drift. According to the structural assumptions by the architects, the outriggers are effective only under live and wind loads as these were installed after the completion of the tower main frame. In the dynamic response, these outriggers are expected to enhance the lateral stiffness and torsional rigidity. The extent to which this is true is studied in FEM51 where the sixteen outrigger frame elements are deleted from FEM5.

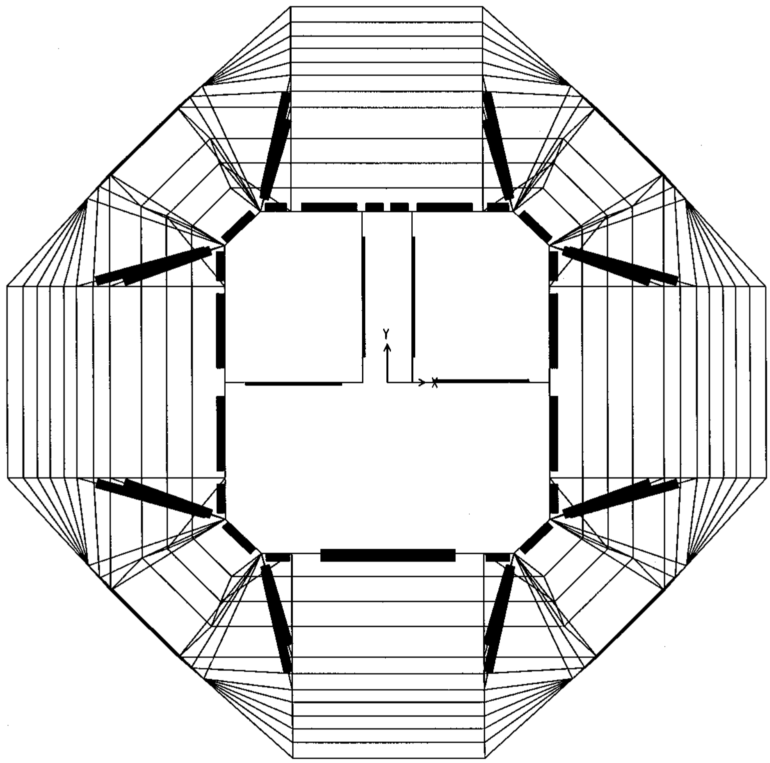

Figure 9. Plan view of FEM5 showing outriggers and thin internal walls.

FEM52-model FEM5 without frames

According to the analysis by Teh and Lai [18], the core wall system is expected to resist approximately 90–95 per cent of the lateral shear in the lower levels. For upper levels, the core wall system is less substantial, and shear resistance is largely due to the steel frame. To check the contribution of the framing system to the dynamic response, all frame members including outriggers, columns, and beams are removed to develop FEM52 from FEM5. There are 2223 shell elements, no frame elements and 14 376 DOF in this model.

RESULTS AND DISCUSSION

The results from the various models are compared with the experimental data in terms of natural frequencies and mode shapes.

Natural frequencies

The analytical modal analysis results for natural frequencies are compared with the field measurement results in Table II. The percentage differences of the natural frequencies obtained

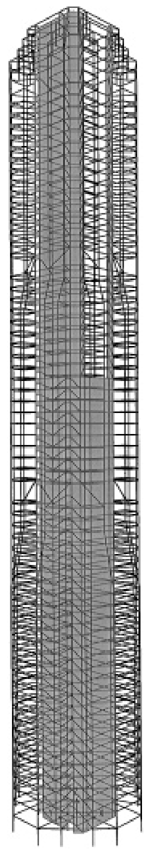

Figure 10. Vertical view of FEM5 with frames.

from the various models with respect to the AVT (experimental) values are listed in Table III. The analytical modal analysis results of FEM5 are most consistent with the experimental results.

Compared with field measurements, it seems that the percentage errors of the stick model (FEM1) for the fundamental modes $X1$ and $Y1$ could be acceptable. However, the other frequencies are too high. This could be in part due to ignoring secondary openings in the core wall, which also has the effect of significantly overestimating the torsional rigidity so that torsional mode frequency $T1$ from FEM1 is 47 per cent too high.

Effect of openings in the core wall

Models FEM2, FEM3, and FEM4 include the complete framing system of columns, beams, and outriggers, but do not consider the contribution of the thin internal walls. The differences among them are in the details of modelling the core wall. Table III shows that the FEM2 closed core wall leads to overestimates in translational modal frequencies by between 13 and 37 per cent while the torsional frequencies are overestimated by over 100 per cent. This indicates that the assumption of closed core wall without openings may give an approximate representation of lateral but not

Table II. Comparison of natural frequencies (Hz).

	FIELD	FEM1	FEM2	FEM3	FEM4	FEM5	FEM51	FEM52
$X1$	0.191	0.170	0.217	0.211	0.190	0.194	0.165	0.142
$X2$	0.703	0.885	0.896	0.586	0.803	0.831	0.799	0.760
$X3$	1.550	2.244	2.124	1.478	1.560	1.700	1.676	1.618
$Y1$	0.199	0.177	0.224	0.224	0.212	0.215	0.182	0.157
$Y2$	0.746	0.897	0.931	0.890	0.814	0.834	0.820	0.807
$Y3$	1.730	2.228	2.209	2.070	1.858	1.909	1.862	1.830
$T1$	0.566	0.830	1.183	0.998	0.489	0.605	0.587	0.555
$T2$	1.340	1.779	3.029	1.896	1.038	1.288	1.263	1.205
$T3$	2.310	2.804			2.055	2.264	2.208	2.129

FIELD field measurements
FEM1 the stick model without frames
FEM2 the model with closed core wall
FEM3 the model with major opening along the x-direction
FEM4 the model with major opening along the x-direction and rows of secondary openings
FEM5 the model with major opening along the x-direction and rows of secondary openings, and internal thin walls
FEM51 the model in FEM5 without outriggers
FEM52 the model in FEM5 without frames

Table III. Percentage errors in natural frequencies from AVT values.

	FEM1 (%)	FEM2 (%)	FEM3 (%)	FEM4 (%)	FEM5 (%)	FEM51 (%)	FEM52 (%)
$X1$	− 11	14	10	− 1	2	− 14	− 26
$X2$	26	27	− 17	14	18	14	8
$X3$	45	37	− 5	1	10	8	4
$Y1$	− 11	13	12	6	8	− 8	− 21
$Y2$	20	25	19	9	12	10	8
$Y3$	29	28	20	7	10	8	6
$T1$	47	109	76	− 14	7	4	− 2
$T2$	33	126	42	− 23	− 4	− 6	− 10
$T3$	21			− 11	− 2	− 4	− 8

torsional rigidity. Going from FEM2 to FEM3 (by modelling the major openings in the X-direction at higher levels), the overestimation of frequencies for X modes drops significantly as expected, while for Y modes the frequency changes are relatively small.

For torsional modes the overestimates are also considerably reduced showing the strong influence of major opening on the torsional rigidity. Nevertheless, FEM3 still overestimates the rigidity of the real building. With the removal of five vertical lines of secondary openings in the core wall (going to FEM4), the lateral frequencies differ from AVT values by no more than 14 per cent, and all except $X1$ are overestimates. For torsional modes they are more substantially reduced resulting in underestimates of up to 23 per cent. All these changes show that FEM4 provides a good approximation to the lateral stiffness of the real building, and that rows of secondary openings weaken the torsional rigidity of the structural system dramatically.

Above level 38 the major opening in the core wall changes it to an open section. The additional three lines of secondary openings make it into several panels of shear walls inter-connected with rows of coupling beams (3.2 or 2.65 m long and 0.7 m deep). In contrast to the 2-D case, there is no formula to estimate the torsional constant of such 3-D coupled shear walls. From the FE analysis, it is clear that greater attention should be paid to the secondary openings as they greatly reduce the torsional stiffness of the core wall.

Internal thin walls

It appears that FEM4 provides good approximations for this building. However, while the translational mode frequencies are generally overestimated, the torsional frequencies are under-estimated. This indicates a need to include certain structural elements which enhance the torsional rigidity but do little for the lateral stiffness. FEM5 differs from FEM4 by the addition of four panels of 200 mm internal thin walls. With the addition of internal walls, the core wall system becomes a partially closed multi-cell section.

Going from FEM4 to FEM5 the torsional frequencies are increased by up to 21 per cent for mode $T1$ while translational frequencies are only slightly affected. For FEM5, except mode $X2$, errors are within a satisfactory 12 per cent.

The effect of the internal thin walls can be explained by the high torsional rigidity of multi-closed cells. The coupled core wall in FEM4 is partially opened by five secondary openings and one major opening per level. The internal thin walls intersect with the core wall making two new enclosed torsionally stiff multi-cell cross-sections, as shown in Figure 9. On the other hand, these internal walls are very close to the geometric centre, so they contribute little to the lateral stiffness.

Outriggers

In order to investigate the contribution of the outriggers to the structural system, 16 diagonal steel beams located in level 28 and level 47 are removed from FEM5 to form FEM51. Both fundamental translational mode frequencies reduce by almost the same amount (16 per cent) but the effect reduces progressively for higher modes. There is a consistent small (2–3 per cent) reduction for the torsional mode frequencies.

It can be concluded that the outriggers enhance the lateral stiffness greatly for the fundamental translational modes, which respond most strongly to wind loads, but contribute little to higher modes which respond to other types of dynamic loads.

The incorporation of outriggers in this structural system couples the core wall and the exterior frame, enhancing the system's ability to resist overturning forces dramatically. Overturning moments of the core and their associated induced deformations can be reduced through the 'reverse' moment applied to the core at each outrigger intersection. This moment is created by the force couple in the exterior columns at the connections to the outriggers.

Frames and core wall

Going from FEM51 to FEM52 shows the effect of removing the entire exterior frame, whose primary function is to support vertical live loads. The largest effect is on the fundamental translational modes $X1$ and $Y1$, reducing by 12–13 per cent, while for torsional modes the effect is

relatively small. With respect to FEM5, the reductions are larger but still most significant for the fundamental translational modes.

Mode shape analysis

Mode shape analysis is conducted by two methods. One is the modal scale factor (MSF) method. The analytical modal displacements are compared with the experimental results, after proper scaling. This method as shown in Equation (3) is therefore also referred to as eigenvector mixing,

$$\mathrm{MSF}(\psi_{\mathrm{a}1}, \psi_{\mathrm{e}1}) = \frac{\{\psi_{\mathrm{a}1}\}^{\mathrm{T}}\{\psi_{\mathrm{e}1}\}}{\{\psi_{\mathrm{a}1}\}^{\mathrm{T}}\{\psi_{\mathrm{a}1}\}} \tag{3}$$

where the subscript 1 indicates that part of the vector corresponding to the degrees of freedom with actual measurements; the subscript a indicates an analytical vector; and the subscript e indicates an experimental vector. The scaled analytical vector, indicated by the subscript 2 can then be derived from Equation (4).

$$\{\psi_{\mathrm{a}2}\} = \mathrm{MSF}(\psi_{\mathrm{a}1}, \psi_{\mathrm{e}1})\ \{\psi_{\mathrm{a}1}\} \tag{4}$$

The MSF-values were calculated for each matched mode pair (a, e) by summing over all measured DOFs, i.e. one DOF in one direction for each level.

The mode shapes must be paired prior to applying this method, and this is also done for the other method, the modal assurance criterion (MAC), which numerically compares all mode shapes in the analytical results with those in the experimental results. MAC is defined as follows:

$$\mathrm{MAC}(\psi_{\mathrm{a}}, \psi_{\mathrm{e}}) = \frac{(\{\psi_{\mathrm{a}}\}^{\mathrm{T}}\{\psi_{\mathrm{e}}\})^2}{(\{\psi_{\mathrm{a}}\}^{\mathrm{T}}\{\psi_{\mathrm{a}}\})(\{\psi_{\mathrm{e}}\}^{\mathrm{T}}\{\psi_{\mathrm{e}}\})} \tag{5}$$

The $\mathrm{a} \times \mathrm{e}$ MAC values are often presented in MAC matrix form **MAC**(a, e) where appropriate mode pairings are indicated by a value approaching unity. Ideally, the MAC matrix is strongly diagonal, as in this study. Since all vertical mode shapes were measured at location 1 (see Figure 1) at every other floor level, corresponding mode shapes at location 1 of the FEM analysis results were extracted from the output data, taking one DOF in one direction for each level as for MSF values.

These two methods were applied to nine paired mode shapes $X1$–$X3$, $Y1$–$Y3$ and $T1$–$T3$, respectively. Figure 11 shows the mode shapes of the field measurements as individual ordinates compared with those of the modal analysis results (FEM5) scaled by MSF, indicating MAC values.

Since the structure is symmetric in the Y-direction, modes $Y1$, $Y2$, and $Y3$ are purely translational, while the remaining modes $X1$, $T1$, $X2$, $T2$, $X3$, and $T3$ are torsionally coupled. Modes $X1$, $X2$, and $X3$ are mainly lateral motion with a degree of torsion that increases with mode number, while modes $T1$, $T2$, and $T3$ are mainly torsional motion. Figure 12 shows deflections of the horizontal framing system at level 65 for mode 7 i.e. $X3$. This movement of this level clearly has an element of torsion, rotating about a point in the negative Y-direction. Likewise, Figure 13 shows mode 6, i.e. $T2$ in which level 65 is seen to be rotating about a location in the positive Y-direction near the core wall. Both effects are consistent with experimental observations.

From Figure 11, it can be shown that mode shapes of analysis results are consistent with those from field measurements; modes $X1$ and $Y1$–$Y3$ are almost perfectly correlated in their own

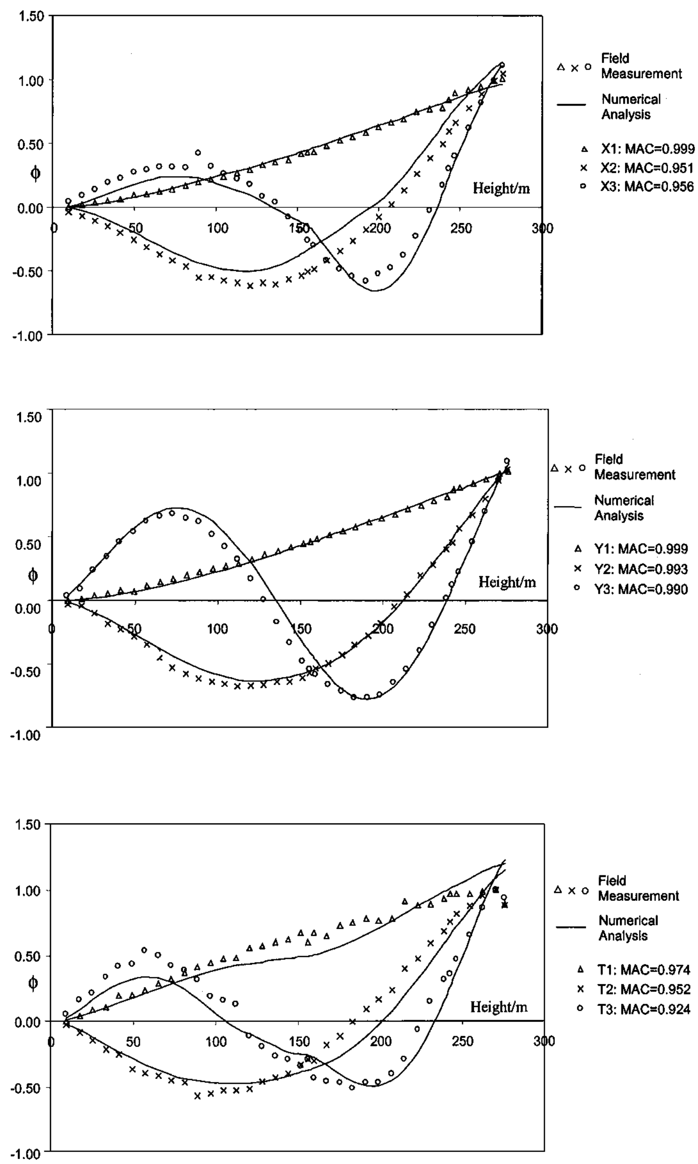

Figure 11. Mode shape correlation analysis of Republic Plaza.

direction. For other modes the MAC values are above 0.924. In general, the MAC values for FEM5 are the highest among all the models.

It is interesting to note that in modes $X3$, $T1$, and $T3$ there is a sharp change in mode shape slope at a height of 151.90 m (the bottom of level 38). It indicates that the lateral stiffness and torsional rigidity of the building drop sharply from this location (level), where the major openings in the core wall are introduced.

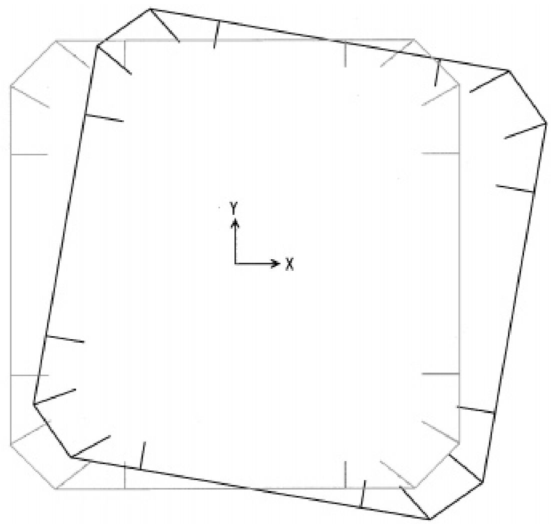

Figure 12. Deflected shape of level 65 framing system for mode $X3$ showing strong torsional component.

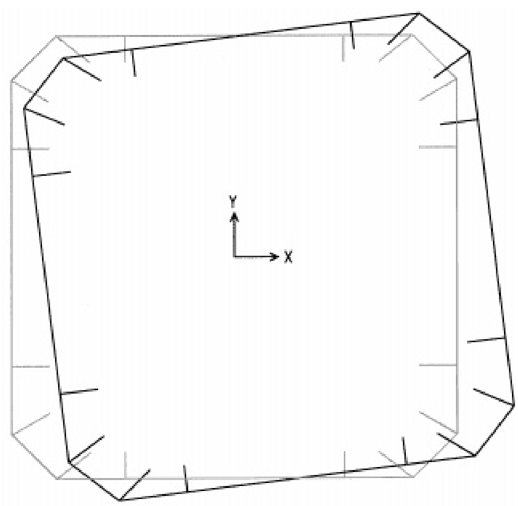

Figure 13. Deflected shape of level 65 framing system for mode $T2$ showing strong rotation about a point near the building perimeter in the positive Y-direction.

CONCLUSIONS

Six 3-D finite element models were built to model the structural system of one tall building, using FE models incorporating lumped masses and rigid floor diaphragms. The natural frequencies and mode shapes obtained by the modal analysis have been analysed and compared with those obtained by the field measurement. The analysis results and comparison show that the following

five factors are very important to correctly model the structural system of a tall building:

1. Outriggers connecting the outer tube with the inner tube make them work together, particularly for simplest translational modes.
2. The stiffening effect of the columns and framing system is largely limited to the fundamental translational modes.
3. The closed core wall overestimates the lateral stiffness, especially the torsional rigidity of the real building.
4. The major opening and vertical rows of secondary openings on the core wall significantly weaken the torsional rigidity of the closed core wall.
5. The thin internal walls within the core should be carefully modeled to enclose part of the open section, which can increase the torsional rigidity of the opened core wall noticeably.

With the rapid development of personal computer (PC) power recently, it is now easier to analyse the structural system by precise FEM with thousands of elements. The difficulty in this field has been shifted from the approximate theoretical analysis to the model updating in FEM analysis. Analytical tools are available [19] to integrate test and analysis. However, there is a restriction to modify certain specific parameter properties such as material properties, geometrical properties, nodal properties, and damping coefficients and it is necessary to have a model such as FEM5 which is already structurally representative and which does not have gross inaccuracy in modal characteristics. The model updating tools cannot usually be used to add or remove complete elements, which should be calibrated according to the structural design concepts. The study of a knowledge-based system for producing the appropriate structurally representative model, which would then be a candidate for conventional updating is a significant topic for future research.

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